SIMULATIONS OF TWO BUBBLES RISE IN CAVITY WITH LATTICE BOLTZMANN METHOD

M. S. Zuhairi*, C. S. N. Azwadi, N. M. Akmal
Universiti Teknologi Malaysia, Faculty of Mechanical Engineering
81310 UTM Skudai, Johor
*zuhairi@ump.edu.my, azwadi@fkm.utm.my

Abstract

This paper explains the research work on two phase flow using Lattice Boltzmann Method. Three type of configuration of the bubbles simulation being simulated using Visual C++ program. Free energy model was reviewed then the base on latest models base on isotropy approach and Galilean invariance are also considered. Simulation results are validated and good agreement with previous studies.

Keywords: Multi-phase, bubbles, lattice Boltzmann method

1. Introduction

Multiphase flow of fluids can be found everywhere either in natural environment phenomenon or in the technology evolution. Study of multiphase flow could contribute a better understanding on multiphase behaviour. The knowledge of multiphase flow behaviour is important in the development of equipment which directly related to multiphase problem.

Lattice Boltzmann Method (LBM) is relatively new method and has a good potential to compete with traditional CFD methods. Recently it has been proved to be a promising tool to simulate the viscous flow [3]. LBM base on derivation of kinetic theory which working in mesoscopic level instead of macroscopic discretisation by traditional method. Instead of easy in incorporating with microscopic physics, it is also having shorter time compare to the current method.

LBM have more advantages in multi-phase compare to traditional method. In multi-phase, two main issues which are surface tension force modelling and interface recording have to be considered [4]

2. Multiphase Lattice Boltzmann Method

Evolution of the equation is important in the initial stage of simulation using Lattice Boltzmann Method. The equation is discretised in time and space, for a set of distribution function f_i. Model of two dimension nine-velocity model (D2Q9) is used evaluate the equation form as

$$f_i(\mathrm{x}+\mathrm{e}_i\Delta t, t+\Delta t) - f_i(\mathrm{x},t) = \frac{1}{\tau}\left[f_i(\mathrm{x},t) - f_i^{eq}(\mathrm{x},t)\right] + F \quad (1)$$

where Δt is time step, e is the particle's velocity, τ is the relaxation time for the collision, F is the external force and $i = 0,1,....,8$. The collision term at the right hand side of Equation 1 has been applied the BGK approximation.

The discrete velocity is expressed as

e_i =(0,0 for i=0,
e_i =cos(i-1)π/4, sin(i-1)π/4 for 1,3,5,7
e_i =$2^{1/2}$ cos(i-1)π/4,sin (i-1)π/4 for i=2,4,6,8.

f_i^{eq} is an equilibrium distribution function, the choice of which determines the physic inherent in the simulation.

The time evolution of the lattice consist of basically two steps which are: a streaming process, where the particle densities area shifted in discrete time steps through the lattice along the connection lines in

direction e_i to their next neighbouring nodes and a collisions step, where locally a new particle distribution is computed by evaluating the right hand side of Equation 1.

Free energy theory in two-phase lattice Boltzmann model states that the physical inherent in simulation described by the equilibrium. Then, a power series in local velocities is assumed.[6].

$$f_i^{eq} = A + B(e_{i,\alpha}u_\alpha) + C(e_{i,\alpha}e_{i,\beta}u_\alpha u_\beta) + Du^2 + G_\alpha \beta e_{i,\alpha}e_{i,\beta} \tag{2}$$

where the summation over repeated Cartesian indices is understood. The coefficient A, B, C, D and $G_{\alpha\beta}$ are determine by placing constraint on the moment of f_i^{eq}. In order that collision term conserves mass and momentum, the first moment of f_i^{eq} are constrained by

$$\sum_i f_i^{eq} = \rho \tag{3}$$

$$\sum_i e_{i,\alpha} f_i^{eq} = \rho u_\alpha \tag{4}$$

The next moment of f_i^{eq} is chosen such that the continuum macroscopic equations approximated by evolution equation correctly describe the hydrodynamics of the component, non-ideal fluid. This gives

$$\sum e_{i,\alpha}e_{i,\beta} f_i^{eq} = P_{\alpha\beta} + \rho u_\alpha u_\beta + \upsilon[u_\alpha \partial_\alpha(\rho) + u_\gamma \partial_\gamma(\rho)\delta_{\alpha\beta}] \tag{5}$$

where $\upsilon = c^2(\tau - 1/2)\Delta t/3$ is the kinematic shear viscosity and $P_{\alpha\beta}$ is the pressure tensor. In order to fully constrain the coeffiecient A, B, C, D and $G_{\alpha\beta}$, a fourth condition is needed, which is

$$\sum e_{i,\alpha}e_{i,\beta} f_i^{eq} = \frac{\rho c^2}{3}(u_\alpha \delta_{\beta\gamma} + u_\beta \delta_{\alpha\gamma} + u_\lambda \delta_{\alpha\beta}] \tag{6}$$

Yonetsu's Approach which is based on isotropic tensor approach has been chosen to the solve the coefficient in the Equation 2-6)

$$A_0 = \rho - 4(A_1 + A_2) - \frac{3}{2c^2}\left[2\upsilon u_\gamma \partial_\gamma \rho + \kappa(\partial_\gamma \rho)^2\right] \tag{7}$$

$$A_1 = 4A_2$$

$$A_2 = \frac{1}{12c^2}[p_0 - \kappa(\partial_\gamma \rho)2 - \kappa\rho\partial_{\gamma\gamma}\rho] \tag{8}$$

$$B_2 = \frac{\rho}{12c^2}, \quad B_1 = 4B_2 \tag{9}$$

$$C_2 = \frac{\rho}{8c^4}, \quad C_1 = 4C_2 \tag{10}$$

$$D_2 = -\frac{\rho}{24c^2}, \quad D_1 = 4D_2, \quad D_0 = -\frac{2\rho}{3c^2} \tag{11}$$

$$G_{2xx} = \frac{1}{8c^4}\left[2\upsilon u_x \partial_x \rho + \kappa(\partial_x \rho)^2\right] \tag{12}$$

$$G_{2xy} = G_{2yx} = \frac{\upsilon}{8c^4}(u_x \partial_y \rho + u_y \partial_x \rho) + \frac{\kappa}{8c^4}(\partial_y \rho) \tag{13}$$

$$G_{2yy} = G_{2xx} \tag{14}$$

$$G_{1\alpha\beta} = 4G_{2\alpha\beta} \text{ for all } \alpha, \beta \tag{15}$$

Analysis by Holdych et al. [5] shows that the evolution scheme, Equation 1 approximates the continuity equations

$$\partial_t \rho + \partial_\alpha(\rho u_\alpha) = 0 \tag{16}$$

and the following Navier-Stokes level equation :

$$\begin{aligned}
&\partial_t(\rho u_\alpha) + \partial_\beta(\rho u_\alpha u_\beta) = \\
&-\partial_\beta P_{\alpha\beta} + \upsilon\partial_\beta[\rho\{\partial_\beta u_\alpha + \partial_\alpha u_\beta + \delta_{\alpha\beta}\partial_\gamma u_\gamma\}] \\
&-\frac{3\upsilon}{c^2}\partial_\beta[u_\alpha \partial_\gamma P_{\beta\gamma} + u\partial_\gamma(\rho u_\alpha u_\beta u_\gamma)] \\
&-\frac{3\upsilon}{c^2}\partial_\beta[(\partial_\beta P_{\alpha\beta})(\partial_\gamma \rho u_\gamma)] \\
&-\frac{3\upsilon}{c^2}\partial_\beta[u_\alpha \partial_\gamma(u_\beta \partial_\gamma \rho + u_\gamma \partial_\beta \rho + \delta_{\gamma\beta}u_\gamma \partial_\gamma \rho)] \\
&-\frac{3\upsilon}{c^2}\partial_\beta[u_\beta \partial_\gamma(u_\alpha \partial_\gamma \rho + u_\gamma \partial_\alpha \rho + \delta_{\alpha\gamma}u_\lambda \partial_\lambda \rho)] \\
&+\frac{3\upsilon}{c^2}\partial_\beta[u_t \partial_\gamma(u_\alpha \partial_\beta \rho + u_\beta \partial_\alpha \rho + \delta_{\alpha\beta}u_\lambda \partial_\lambda \rho)]
\end{aligned} \tag{17}$$

The top line is the compressible Navier-Stokes equation while the subsequent lines are error terms. We have, then describe a framework for one component free energy lattice Boltzmann.

Van-Der Waals theory is very closed related to the multiphase phenomena. The Van-Der Waals equation of state can be describes as follow

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \tag{18}$$

where n is mole number, a and b are constant characteristic of a particular gas and R is the gas constant. p, V and T denotes as volume and temperature. Equation 18 then rewrite in simplified form as follow :

$$\left(\tilde{p}+\frac{3}{\tilde{V}}\right)(3\tilde{V}-1)=8\tilde{T} \tag{19}$$

where

$$V_c=3b,\quad T_c=\frac{8a}{27bR},\quad P_c=\frac{8a}{27b^2} \tag{20}$$

and

$$\tilde{P}=\frac{P}{P_c},\quad \tilde{V}=\frac{V}{V_c},\quad \tilde{T}=\frac{T}{T_c} \tag{21}$$

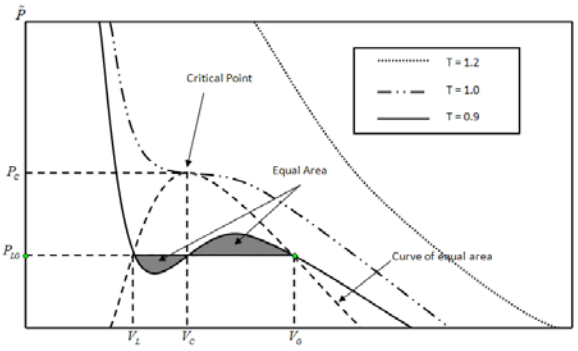


Figure 1: Isotherms plot of $\tilde{p}-\tilde{V}$

The thermodynamics of the fluid enters the lattice Boltmann simulation via pressure tensor $P_{\alpha\beta}$. The equilibrium properties of a system with no surface (i.e periodic boundaries) can be describe by a Landau free energy functional

$$\psi=\int dV\left[\psi(\rho,t)+\frac{\kappa}{2}(\partial_\alpha\rho)^2\right] \tag{22}$$

Subject to constraint

$$M=\int dV\,\rho \tag{23}$$

where $\psi(\rho,t)$ is free energy density of bulk phase. κ is a constant related to the surface tension, M is the total mass of fluid and the integration are over all space. The second term in Equation 22 gives the free energy distribution from density gradients in an homogeneous system. For Van-Der Waals fluid, free energy density of bulk phase can be written in the form

$$\psi(\rho,t)=\rho RT\ln\left(\frac{\rho}{1-b\rho}\right)-a\rho^2 \tag{24}$$

Introducing a constant Lagrange multiplier, μ, we can minimise Equation 22, giving a condition for equilibrium as

$$\frac{\partial\psi}{\partial\rho}-\mu-\kappa\nabla^2\rho=0 \tag{25}$$

By multiplying Equation 25 by $\frac{\partial\rho}{\partial x}$ and integrating once with respect to x, we obtain first integral

$$\psi-\mu\rho-\frac{\kappa}{2}\left(\partial_\alpha\rho\right)^2=\text{constant} \tag{26}$$

At equilibrium condition, the chemical potential and pressure of both phases are given by

$$\mu=RT\ln\left(\frac{\rho}{1-b\rho}\right)+\frac{RT}{1-b\rho}-2a\rho \tag{27}$$

$$p=\frac{\rho RT}{1-b\rho}-2a\rho^2 \tag{28}$$

Respectively. We now define $W(\rho,T)=\psi-\mu\rho+p$, then the Equation 25 and Equation 26 can be rewritten as

$$\frac{\partial W}{\partial\rho}=\kappa\nabla^2\rho \tag{29}$$

$$W=\frac{\kappa}{2}\left(\partial_\alpha\rho\right)^2 \tag{30}$$

By solving equation 30, we can determine the density profile at the interface for different values of κ .Noted that the fourth order Rungge-Kutta scheme is used to solve equation 30 and temperature is set to be $T = 0.55$.

The Eotvos equation stated that the increasing of Eo number are parallel with the decreasing of the surface tension. By decreasing of the σ , it will enhance the deformation of the bubbles.

$$Eo = \frac{g\Delta\rho d^2}{\sigma} \quad (31)$$

$$Mo = \frac{g\rho_L \Delta\rho \upsilon^4}{\sigma^3} \quad (32)$$

$$\mathrm{Re} = \frac{Ud}{\upsilon} \quad (33)$$

where g is the gravitational force, $\Delta\rho$ is the density difference for two phase system, ρ_L is the fluid density, U is the velocity at equilibrium state, d is the radius of bubble and σ is the surface tension coefficient.

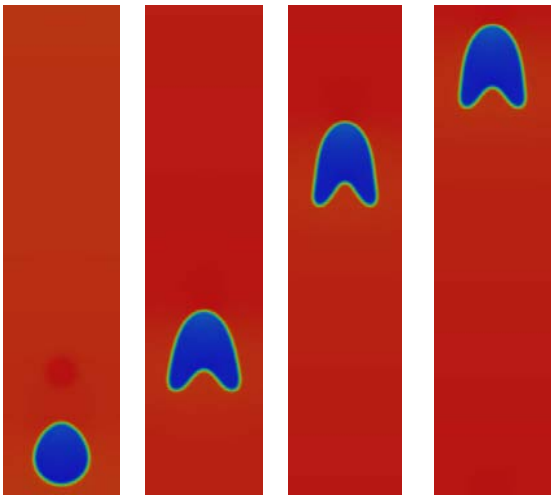
3. Results and Discussions

Simulation of two bubbles rise was done for three different cases where several parameters were varied.

In these simulations, density is set to be 0.0001962, κ value is set to be 0.0075, density of fluid is set to 4.895 for liquid and 2.211 for gas. The periodic boundary condition is employed in all boundaries. In this study, the cavity shaped grid was used. For Case 1 and 2, grid size of 161 x 481 was used while for Case 3, grid of 201 x 681 was used.

The simulation for Case 1 and Case 2 used the same Eotvos number which is Eo =20 while Case 3 used Eo = 30. Due to the effect of the buoyancy force, the bubbles moved upward from the bottom and in the meantime the bottom-middle part of the bubbles experienced a deformation due to hitting process with the surrounding water.

3.1. Case 1

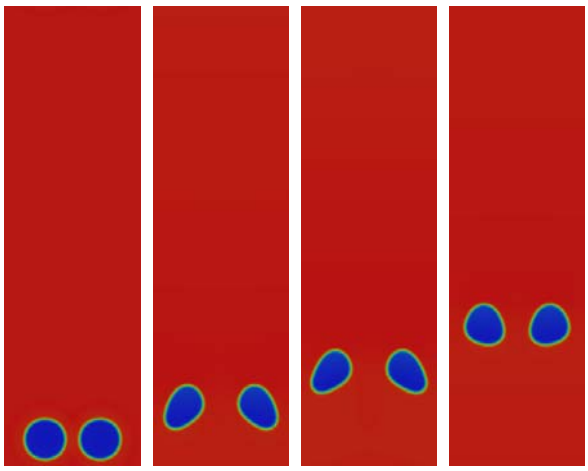


(a) (b) (c) (d)

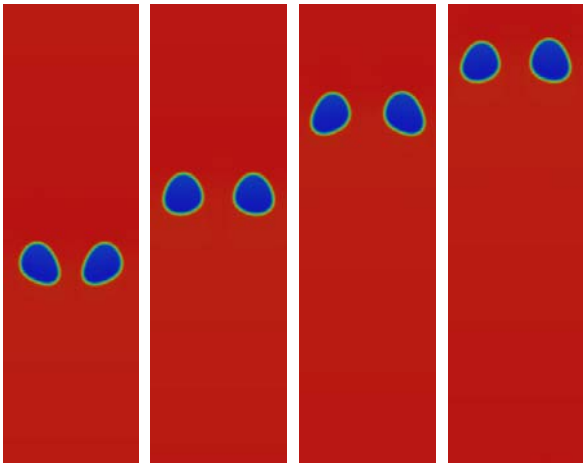
Figure 2 (a) – (d): Time evolution of bubble rise phenomenon at Eo =20

The figures show the bubble deformation when the bubble arose upward from the bottom. The bubble shape deform accordingly due to the surface tension force while moving upwards due to the buoyancy effect. The interfaces of bubble and water are having surface tension force. The formation of wake below the bubble causes it to take the form of a skirt.

3.2. Case 2



(a) (b) (c) (d)

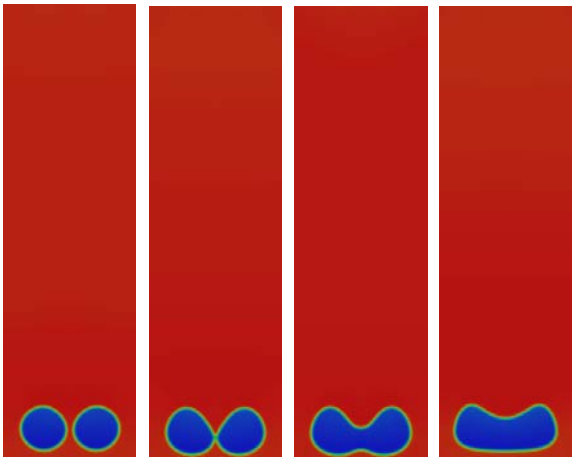


(e) (f) (g) (h)

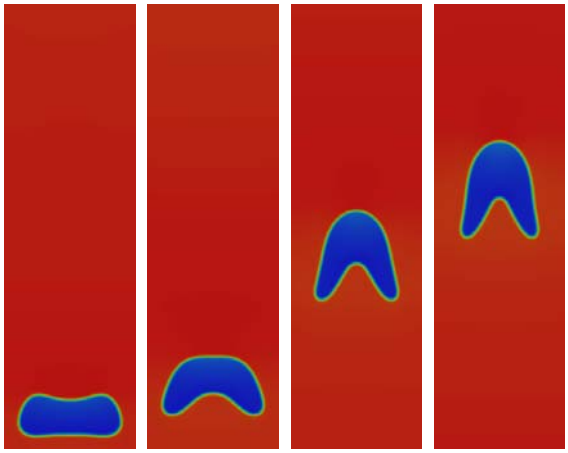
Figure 3 (a) – (h): Time evolution of bubble rise phenomenon at Eo=20

Figures shows two separate bubbles positioned side by side area configured relatively far to the each others. As the time step increase, the shape of the bubbles deform due to surface tension force which take an oblate spheroid shape. The phenomenon was resulted by spurious current upward in the middle of both bubbles. The bubbles are not merged each other due to the interface thickness stronger than surface tension force. Lower Eotvos number resulted stronger interface.

3.3. Case 3



(a) (b) (c) (d)



(e) (f) (g) (h)

Figure 4 (a) – (h): Time evolution of bubble rise phenomenon at Eo=30

The figures explain the two bubbles coalesce each other when they were located relatively closed. By increasing Eotvos number to 30 means lowering the interface strength. Later it contributed to the merging effect of the bubbles. The interface force was unable to resist the surface tension resulted by the spurious current and eventually starting to merge. At the meantime of merging process, the bubble continues moving upward due to buoyancy effect.

4. Conclusion

This paper shows the capability LBM in simulating the multi-phase phenomenon. Several simulations of bubbles rise has been done. From simulations, Case 2 and Case 3 bring an understanding on the interface force that reacts with the surface tension force of the bubbles movement. It was clear that the bubbles in the Case 2 not affected with the surface tension forces from each other. For the Case 3 which is configured close to each other, the bubbles later merge to each other due to the broken interface cause by surface tension force. The result shows the LBM capability in simulating the multi-phase flow. The study has been compared to the previous study and having a good agreement with other researcher.

5. Acknowledgements

The authors wish to thank Universiti Teknologi Malaysia and Universiti Malaysia Pahang for supporting this research activity.

6. References

[1] Nor Azwadi Che Sidik, Takahiko Tanahashi, Two-phase Flow Simulation with Lattice Boltzmann Method, *Jurnal Mekanikal December 2007, No. 24, 68 - 79*

[2] R. Zhang , X. He, S. Chen, Interface and Surface Tension in Incompressible Lattice Boltzmann Multiphase Model, *Computer Physics communications 129 (2000) 121–130*

[3] S. Chen, G. Doolen, Lattice Boltzmann method for fluid flows, *Annu. Rev. Fluid Mech. 30 (1998) 329–364.*

[4] H.W. Zheng, C. Shu , Y.T. Chew, A lattice Boltzmann model for multiphase flows with large density ratio, *Journal of Computational Physics 218 (2006) 353–371.*

[5] Holdych, D.J., Geogiadis, J.G., Buckius, R.O., 2001. Migration of a Van Der Waals Bubbles: Lattice Boltzmann Formulation, *Physics of Fluids 13, 817-825.*

[6] Gunstensen, A.K., Rothman, D.H., Zaleski, G., 1991. Lattice Boltzmann Model for Immiscible Fluids, *Physics Review A 43, 4320-4327.*